Research Paper | Mathematics | E-ISSN No : 2454-9916 | Volume : 4 | Issue : 11 | Nov 2018

ON THE CULTIVATION OF STUDENTS' COMPREHENSIVE ABILITY IN ANALYTICAL GEOMETRY TEACHING

Wang Yutian

School of Mathematics and Statistics, Shandong University of Technology, Zibo, China.

ABSTRACT

In this paper, by using the viewpoint of vector in analytic geometry course, a method of solving three-dimensional homogeneous linear equations is given, and how to cultivate students' ability to apply knowledge comprehensively in the teaching of analytic geometry is expounded in detail.

KEYWORD: Analytic geometry, linear equations, comprehensive application, innovation.

Analytical geometry is a branch of mathematics that uses algebraic method to study spatial geometric figure. It is one of the important basic courses for college mathematics. As far as teaching is concerned, analytic geometry provides guiding ideology for mathematics education and contains many different mathematical ideas and methods. Therefore, in the teaching of analytic geometry, we should persist in guiding students to observe, discover and contact, and strive to cultivate students' ability of knowledge transfer and comprehensive application, so as to achieve analogy and achieve good teaching results. Combined with my own teaching practice, I will talk about the experience.

1. **Cultivating students' knowledge transfer ability through curriculum integration**

Higher algebra is an important basic course for college mathematics majors. Solving linear equations is an important part of higher algebra. In the teaching of analytic geometry, students are guided to reconsider ternary homogeneous linear equations from the point of view of geometric vectors. Through the integration of knowledge of two courses, students' knowledge transfer ability and innovation consciousness can be cultivated.

The geometric Significance of linear equations:

$$Ax + By + Cz = 0 \tag{2}$$

When , B, C are not all zero (let's say $A \neq 0$), $Ax + By + Cz = 0$ represents a plane equation in space, then the solution set of equation (1) is the set of all points on the plane $Ax + By + Cz = 0$.

The geometric Significance of linear equations

$$\begin{cases} A_1x + B_1y + C_1z = 0 \\ A_2x + B_2y + C_2z = 0 \end{cases} \tag{3}$$

When the coefficients of two equations are not all zero (let's say $A_1 \neq 0, A_2 \neq 0$), the solution set of the system (2) is the set of all points perpendicular to the plane $A_ix + B_iy + C_iz = 0$ (i=1,2).

The geometric Significance of linear equations

$$\begin{cases} A_1x + B_1y + C_1z = 0 \\ A_2x + B_2y + C_2z = 0 \\ A_3x + B_3y + C_3z = 0 \end{cases} \tag{4}$$

When the coefficients of the equations are not all zero, the solution set of the equations (3) is the set of all points perpendicular to the plane $A_ix + B_iy + C_iz = 0$ (i=1,2,3).

2. **Cultivate students' comprehensive ability through analysis and discussion:**

According to the above analysis of ternary homogeneous linear equations from the vector point of view, the students are guided to further study the solution of the equations by using the knowledge of vector operation comprehensively, so as to cultivate the students' logical reasoning ability and comprehensive application of knowledge.

Example 1: Solving linear equations

$$Ax + By + Cz = 0 \tag{1}$$

Suppose $\vec{r} = (x, y, z)$, $\vec{u} = (A, B, C)$, then

$Ax + By + Cz + D = 0 \Leftrightarrow \vec{r} \cdot \vec{u} = 0$.

Take two non collinear vectors perpendicular to $\vec{u}$:

then $\vec{r} = \lambda\vec{r_1} + \mu\vec{r_2}$ (λ, μ are arbitrary real numbers) is the solution of linear equation (1).

Example 2: Solving linear equations

$$\begin{cases} A_1x + B_1y + C_1z = 0 \\ A_2x + B_2y + C_2z = 0 \end{cases} \tag{2}$$

Suppose $\vec{r} = (x, y, z)$, $\vec{u} = (A_1, B_1, C_1)$, $\vec{v} = (A_2, B_2, C_2)$, then

$$\begin{cases} A_1x + B_1y + C_1z = 0 \\ A_2x + B_2y + C_2z = 0 \end{cases} \Leftrightarrow \begin{cases} \vec{u} \cdot \vec{r} = 0 \\ \vec{v} \cdot \vec{r} = 0 \end{cases}.$$

Case 1. If $\vec{u}$ is in collinear with $\vec{v}$ May as well set up $\vec{v} = d\vec{u}$. Take two non collinear vectors perpendicular to $\vec{u}$:

$$\vec{r_1} = \left(-\frac{B_1}{A_1}, 1, 0\right), \vec{r_2} = \left(-\frac{C_1}{A_1}, 0, 1\right)$$

then $\vec{r} = \lambda\vec{r_1} + \mu\vec{r_2}$ (λ,μ are arbitrary real numbers) is the solution of linear equation (2).

Case 2. If $\vec{u}$ is not collinear with $\vec{v}$, then

$$\vec{r} = t(\vec{u} \times \vec{v}) = t\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = t\left(\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}, \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix}, \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}\right)$$

is the solution of linear equation (2).

Example 3: Solving linear equations

$$\begin{cases} A_1x + B_1y + C_1z = 0 \\ A_2x + B_2y + C_2z = 0 \\ A_3x + B_3y + C_3z = 0 \end{cases} \tag{3}$$

Suppose $\vec{r} = (x, y, z), \vec{u} = (A_1, B_1, C_1), \vec{v} = (A_2, B_2, C_2), \vec{w} = (A_3, B_3, C_3)$, then

$$\begin{cases} A_1x + B_1y + C_1z = 0 \\ A_2x + B_2y + C_2z = 0 \\ A_3x + B_3y + C_3z = 0 \end{cases} \Leftrightarrow \begin{cases} \vec{u} \cdot \vec{r} = 0 \\ \vec{v} \cdot \vec{r} = 0 \\ \vec{w} \cdot \vec{r} = 0 \end{cases}$$

Case 1. If $\vec{u}$ does not coexist with $\vec{v}$, $\vec{w}$, then $\vec{r} = \vec{0}$.

Case 2. If $\vec{u}$ coexists with $\vec{v}$, $\vec{w}$, and $\vec{u}$, $\vec{v}$, $\vec{w}$ do not collinear each other.

Let's take just $\vec{w}$ to be $\vec{w} = \lambda\vec{u} + \mu\vec{v}$, then

$\vec{v}$

$$\vec{r} = t(\vec{u} \times \vec{v}) = t\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = t(\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}, \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix}, \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix})$$

is the solution of linear equations (3).

Case 3. If $\vec{u}$ coexists with $\vec{v}$, $\vec{w}$, and there are only two vectors collinear.

Let's take just $\vec{w}$ to be $\vec{w} = \lambda\vec{v}$, then

$$\vec{r} = t(\vec{u} \times \vec{v}) = t\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = t(\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}, \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix}, \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix})$$

is the solution of linear equations (3).

Case 4. If $\vec{u}$, $\vec{v}$, $\vec{w}$ are collinear. May as well set up $\vec{v} = \lambda\vec{u}, \vec{w} = \mu\vec{u}$, The original equations are equivalent to $\vec{r} \cdot \vec{u} = 0$. Take two non collinear vectors perpendicular to $\vec{u}$

$$\overrightarrow{v_1} = (-\frac{B}{A}, 1, 0), \overrightarrow{v_2} = (-\frac{C}{A}, 0, 1)$$

then $\vec{r} = \lambda\overrightarrow{v_1} + \mu\overrightarrow{v_2}$ (λ,μ are arbitrary real numbers) is the solution of linear equation (3).

3. Summary:

In the teaching of analytic geometry, the core idea of cultivating students' comprehensive application ability is to help students form the habit of thinking divergence, through the interrelation of knowledge, use knowledge comprehensively and solve problems. Teachers must take this as a breakthrough, choose appropriate teaching materials, effectively organize teaching, and guide students to think more. This is conducive to students' cognitive coherence, more flexible thinking, wider vision; more conducive to improving students' cognitive development level, improve students' comprehensive use of knowledge to solve problems and innovative ability, thus effectively promoting the teaching of analytical geometry.

REFERENCES:

1. H.B. Griffith. Comprehensive textbook of classical mathematics [M]. Guizhou people's Publishing House, 1986
2. Gong AI Fang, Guo Mingyue. Discussion and Reflection on various solutions of linear equations. Journal of [J] higher continuing education. 2005:41-44.
3. Lv Lingen, Xu Zi Dao. Analytical geometry Fourth Edition [M]. higher education press, 2006.